ELSEVIER

Int. J. Therm. Sci. 41 (2002) 376–385

International Journal of Thermal Sciences

www.elsevier.com/locate/ijts

Effects of backward- and forward-facing steps on turbulent natural convection flow along a vertical flat plate

H.I. Abu-Mulaweh

Mechanical Engineering Department, Purdue University at Fort Wayne, 2101 E. Coliseum Blvd., Fort Wayne, IN 46805, USA

Received 30 January 2001; accepted 7 June 2001

Abstract

The effects of backward-facing and forward-facing steps on turbulent natural convection along a vertical heated flat plate are examined experimentally. Laser-Doppler velocimeter and cold wire anemometer were used to, respectively, measure simultaneously the time-mean turbulent velocity and temperature distributions and their turbulent fluctuation intensities. The experiment was carried out for a step (backward-facing and forward-facing) height of 22 mm and a temperature difference between the heated walls and the free stream (ambient air), ΔT, of 30 °C (corresponding to a local Grashof number $Gr_{xi} = 6.45 \times 10^{10}$). Measurements of the time-mean velocities and temperature distributions along with their turbulent fluctuation intensities are presented at various locations downstream from the location of the step. The present results reveal that the maximum local Nusselt number occurs in the vicinity of the reattachment region and it is approximately twice for the case of backward-facing step and two and a half times for the case of forward-facing step, than that of the flat plate value at similar flow and thermal conditions.

Keywords: Heat transfer; Separated flow; Turbulent; Convection; Natural

1. Introduction

The existence of flow separation and subsequent reattachment due to a sudden compression or expansion in flow geometry, such as forward-facing and backward-facing steps, respectively, plays an important role in the design of many engineering applications where cooling or heating is required. These heat transfer applications appear in cooling systems for electronic equipment, combustion chambers, chemical processes and energy system equipment, environmental control systems, high performance heat exchangers, and cooling passages in turbine blades. A significant amount of mixing of high and low energy fluid occurs in the reattached flow region in these devices, thus affecting their heat transfer performance. For the flow past a backward-facing step, the flow separates at the upper sharp corner of the step causing a recirculating region to develop behind the step. On the other hand, for the flow past a forward-facing step, one or two separated regions may develop, one upstream and the other downstream from the step, depending on the ratio of the boundary-layer thickness at the step to the step height, as reported by Abu-Mulaweh et al. [1]. The problem of laminar flow over backward-facing and forward-facing step geometries in natural, forced, and mixed convection has been investigated extensively in the past, both numerically and experimentally (Lin et al. [2], Hong et al. [3], and Abu-Mulaweh et al. [4,5] and the references cited therein). Also, many investigators examined turbulent forced convection flow over a backward-facing step (Vogel and Eaton [6], Abe et al. [7], Rhee and Sung [8] and the references cited therein). Turbulent natural convection flow over a vertical backward-facing step was recently examined by Inagaki [9] and Abu-Mulaweh et al. [10]. In the present study, the effects of both backward-facing and forward-facing steps on the flow and thermal fields characteristics of turbulent natural convection flow over a vertical flat plate are examined experimentally.

The step geometries consist of an upstream wall, a backward-facing step or forward-facing step and a downstream wall. The flat plate geometry and both the upstream and downstream walls of the two step geometries were heated to a constant and uniform temperature. Measured time-mean velocity and temperature, intensities of velocity and temperature fluctuations, reattachment length and local Nusselt number distributions are presented.

E-mail address: mulaweh@engr.ipfw.edu (H.I. Abu-Mulaweh).

1290-0729/02/$ – see front matter
PII: S1290-0729(02)01328-5

Nomenclature

Gr_x	local Grashof number $= g\beta(T_w - T_\infty)x^3/\nu^2$	
g	gravitational acceleration	$m{\cdot}s^{-2}$
h	local heat transfer coefficient $= -k(\partial T/\partial y)_{y=0}/(T_w - T_\infty)$	$W{\cdot}m^{-2}{\cdot}{}^\circ C^{-1}$
k	thermal conductivity	$W{\cdot}m^{-1}{\cdot}{}^\circ C^{-1}$
Nu_{x^*}	local Nusselt number $= hx^*/k$	
s	step height	mm
T	fluid temperature	°C
T_∞	ambient temperature	°C
T_W	wall temperature	°C
$\overline{t'^2}$	intensity of temperature fluctuations	$°C^2$
u	mean streamwise velocity	$m{\cdot}s^{-1}$
u^*	reference velocity $= [g\beta(T_w - T_\infty)x_i]^{1/2}$	$m{\cdot}s^{-1}$
$\overline{u'^2}$	intensity of streamwise velocity fluctuations	$m^2{\cdot}s^{-2}$
U	dimensionless mean streamwise velocity $= u/u^*$	
v	mean transverse velocity	$m{\cdot}s^{-1}$
$\overline{v'^2}$	intensity of transverse velocity fluctuations	$m^2{\cdot}s^{-2}$
V	dimensionless mean transverse velocity $= v/u^*$	
x, y	streamwise and transverse coordinates measured from the downstream plate	m
x^*	$= x + x_i$	m
x_i	inlet length upstream of the step	m
x_r	reattachment length	cm
Y	$= y/x^*$	

Greek symbols

η	similarity variable $= (y/x)Gr_x^{1/4}$	
β	coefficient of thermal expansion	K^{-1}
ΔT	temperature difference $= (T_w - T_\infty)$	°C
θ	dimensionless temperature $= (T - T_\infty)/(T_w - T_\infty)$	
ν	kinematic viscosity	$m^2{\cdot}s^{-1}$

2. Experimental apparatus and procedure

The experimental investigation was carried out in an existing low turbulence, open circuit air tunnel that was oriented vertically. Details of the air tunnel have been described by Abu-Mulaweh et al. [10] and a schematic is shown in Fig. 1. It had a smooth converging nozzle, a straight square test section, and a smooth diverging diffuser. The tunnel was constructed from a 1.27 cm thick plexiglass plate and a 1.91 cm thick plywood sheet with steel frames and supports to provide a rigid structure. The test section, which was constructed from transparent plexiglass material, allows flow visualizations and permits the use of a laser-Doppler velocimeter for velocity measurements. The flat plate and both step geometries were supported (one of them at a given time) in the test section of the tunnel and span its entire width (85.1 cm). A cross section of 63.5 cm by 85.1 cm was provided adjacent to the test surface in the test section for the developing boundary layer air flow.

Fig. 2 is a schematic diagram of the flat plate geometry and the two steps geometries. Both of the steps geometries consist of a backward-facing step or forward-facing step (2.2 cm in height), an upstream wall (274.3 cm in length), and a downstream wall (81.3 cm in length). The flat plate geometry and both the upstream and the downstream walls of the two steps geometries were heated to a constant and uniform temperature. The heated walls were made of three composite layers that were held together by screws. The upper layer was an aluminum plate (85.1 cm wide, and 1.27 cm thick) instrumented with several thermocouples that were distributed in the axial direction along its entire length. Each thermocouple is inserted into a small hole from the backside of the plate and its measuring junction is flush with the test surface. The middle layer consists of several heating pads that can be controlled individually for electrical energy input. By controlling the level of electrical energy input to each of the heating pads, and monitoring the local temperature of the heated walls with the imbedded thermocouples, the temperature of the heated surface can be maintained constant and uniform to within 0.2 °C. The bottom layer of the heated walls is a 1.91 cm thick plywood board serving as backing and support for the heated wall structure. The front edge of the upstream plate is chamfered to insure a proper development of the boundary layer flow.

Air temperature and velocity were measured simultaneously by using, respectively, a cold wire anemometer and a two-component laser-Doppler velocimeter (LDV). The LDV system is equipped with an automated three-dimensional traverse system for positioning the measuring LDV probe volume at any desired point in the flow domain. The cold wire probe with a separate traverse system is placed within 2 mm behind the measuring LDV volume. The outputs from both the LDV and the cold wire anemometer are then processed through an A/D converter and a suitable software on an IBM personal computer to determine the local instantaneous velocity and temperature. These measurements were then used to determine the time-mean velocity and temperature, intensities of velocity and temperature fluctuations, and local Nusselt number distributions.

In this experimental study, the free stream (ambient air) temperature was not constant along the length of the plate. It increased by approximately 3 °C along the length of the entire heated wall(s) (equivalent to $0.9\ {}^\circ C{\cdot}m^{-1}$). This should be expected because the air flow is confined in the tunnel

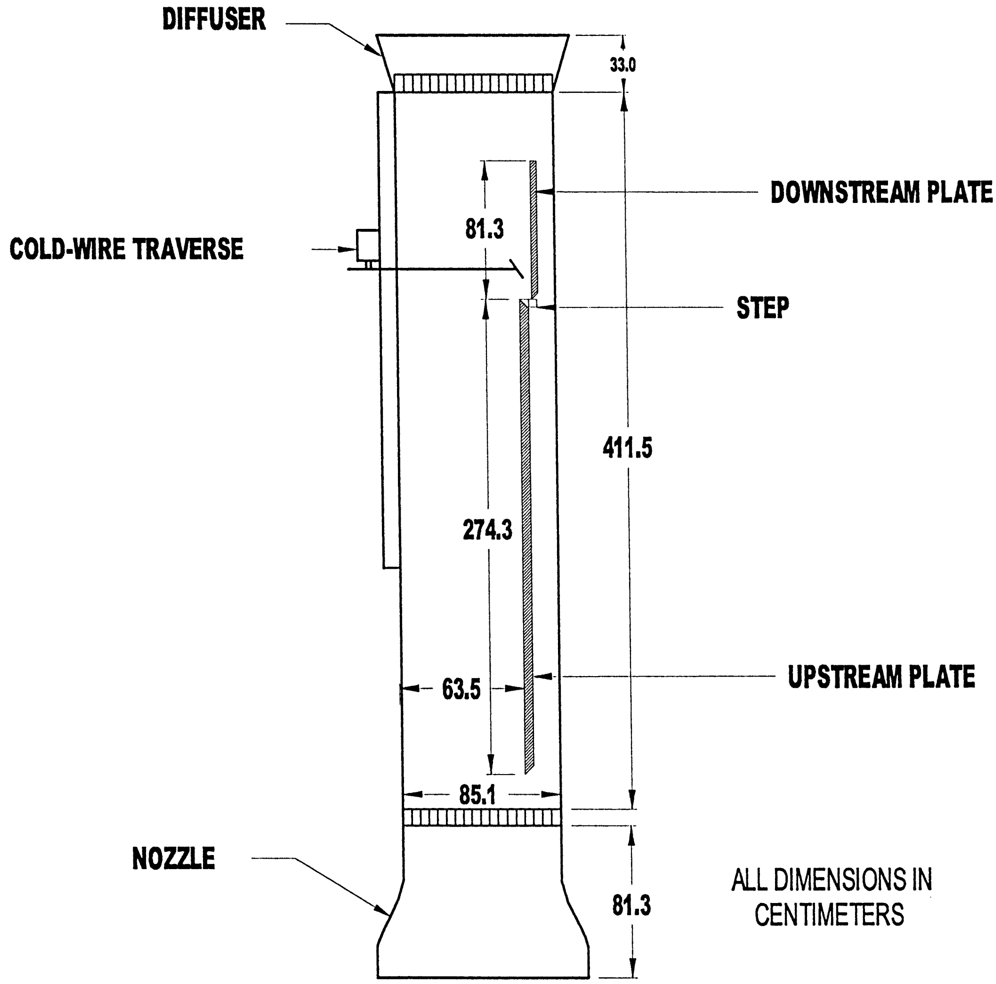

Fig. 1. Schematics of the air tunnel.

boundaries (a finite domain as compared to the infinite domain normally assumed in numerical modeling of natural convection adjacent to a vertical heated plate) and energy is continually being added to the heated walls. Some of that energy flows to the free stream, thus causing its temperature to increase slightly along the length of the plate.

It was established, through repeated LDV measurements, that 1024 acceptable LDV samples of the local instantaneous fluctuating velocity component were sufficient to repeatedly and accurately determine the local mean velocity and temperature in the flow domain. The acceptable sampling rate for these measurements varied between 10 and 100 sample$\cdot$sec^{-1}. All of the reported data in this study resulted from taking the average of two separate measurements taken back to back, each of which having a sample of 1024 instantaneous measurements.

The repeatability of the mean velocity measurements was determined to be within 4%, and that of the temperature measurements was within 0.25 °C (0.5%). The uncertainties in the measured results were estimated (at the 95% confidence level) according to the procedure outlined by Moffat [11] and they are reported in the appropriate section of this paper.

Flow visualizations were also performed to verify the boundary-layer development and its two-dimensional nature. These flow visualizations were carried out by using a 15-Watt collimated white light beam, 2.5 cm in diameter. Glycerin smoke particles, 2 to 5 microns in diameter, which are generated by immersing a 100 Watt heating element into a glycerin container, are added to the inlet air flow and used as scattering particles for flow visualization and for LDV measurements.

3. Results and discussion

The air flow boundary layer development in the experimental set up, along with its two-dimensional nature, was verified through flow visualization and through measurements of velocity and temperature across the width of the tunnel, at several heights above the heated wall. These measurements showed a wide region (65 cm or about 80% of the width of the heated wall around its center $z = 0$) where the air flow velocity and temperature distributions, in the z direction at a fixed distance from the heated wall, could be approximated (to within 5%) as uniform and two-dimensional

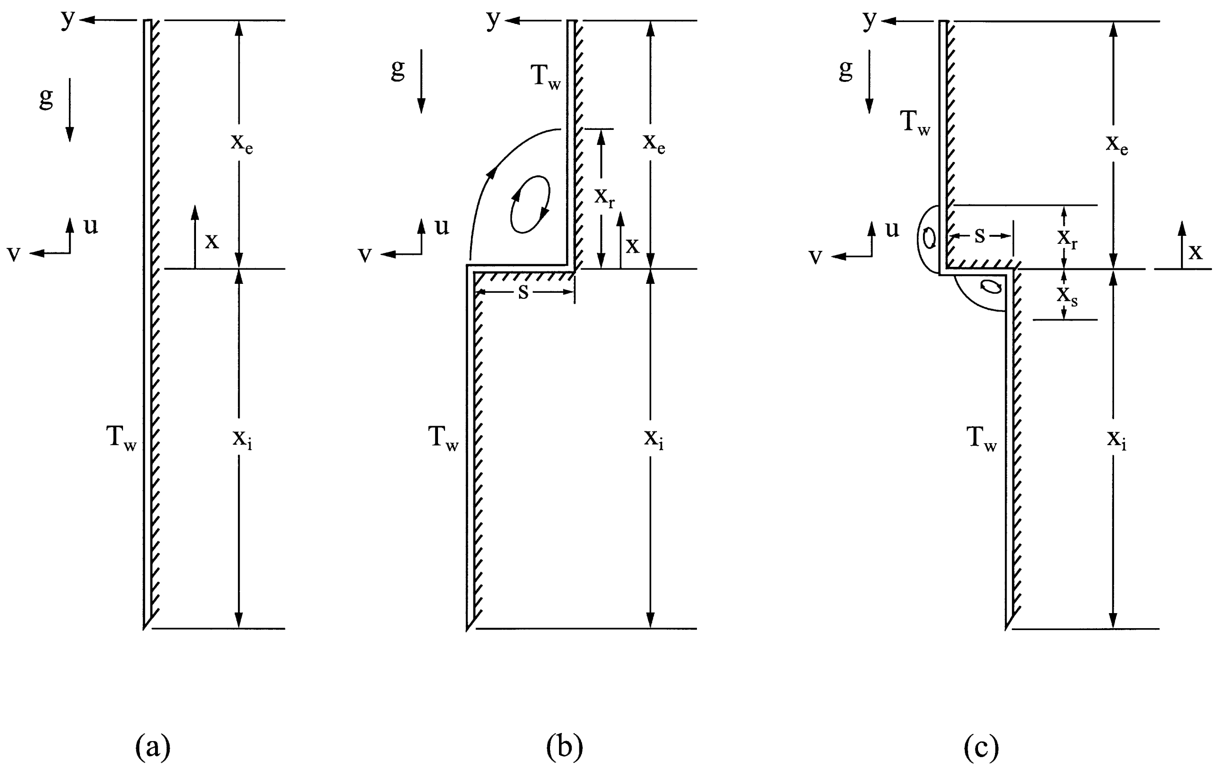

Fig. 2. Schematics of the flow geometry, (a) flat plate, (b) backward-facing step, (c) forward-facing step.

flow. In addition, velocity and temperature measurements were performed at the center of the tunnel's width ($z = 0$) and normal to the heated wall, at various locations, to determine the conditions in the mainstream. These measurements revealed that the effects of the steps and the thermal conditions of the wall on the mainstream diminish at distances beyond ten step heights (22 cm) in the transverse direction away from the heated wall. Velocity and temperature measurements in the region of 10–20 step heights (22–44 cm) in the transverse direction away from the heated wall, displayed a mainstream flow region with uniform velocity and temperature distributions that are equivalent to the free-stream condition of a natural convection boundary layer flow. Also, the flow adjacent to the unheated wall of the tunnel (wall opposite to the heated wall) was visualized to establish the existence of reversed flow in that region. These observations established that a reverse flow existed in the upper part of the tunnel in a small and very narrow region adjacent to the unheated wall. That region did not penetrate into the domain where measurements were made.

The operation of the air tunnel, its instrumentation, along with the accuracy and the repeatability of the measurements were validated by performing measurements of turbulent natural convection boundary-layer flow adjacent to an isothermal vertical heated flat plate. Both the measured flow and thermal fields for this case compared well with other previously measured results. Fig. 3 provides a comparison of the mean streamwise velocity between the present measurements and those of Cheesewright [12] and the numerical predictions of Plumb and Kennedy [13]. The mean

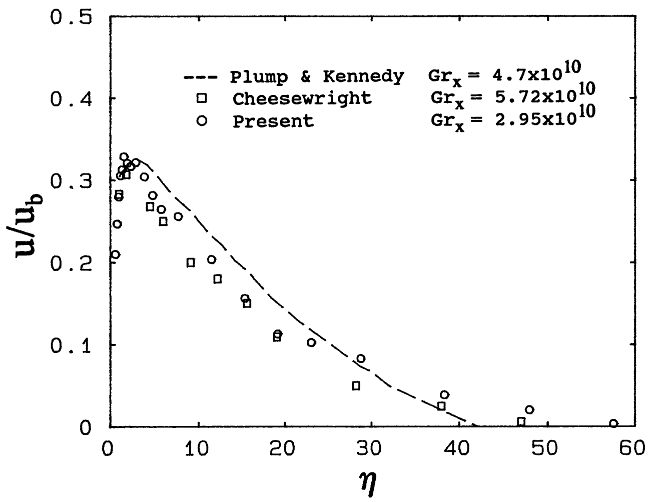

Fig. 3. Dimensionless mean streamwise velocity distribution adjacent to flat plate.

streamwise velocity is normalized with the buoyant velocity, $u_\mathrm{b} = (g\beta\Delta T x)^{1/2}$. The good agreement between the results in the figure validates the performance of the air tunnel and its instrumentation. All reported measurements were taken along the midplane ($z = 0$) of the plate's width, and only after the system had reached steady state conditions.

To examine the effects of backward-facing and forward-facing steps on the flow and the thermal fields of turbulent natural convection flow along a vertical heated flat plate, measurements of velocity and temperature distributions were carried out at three different streamwise locations

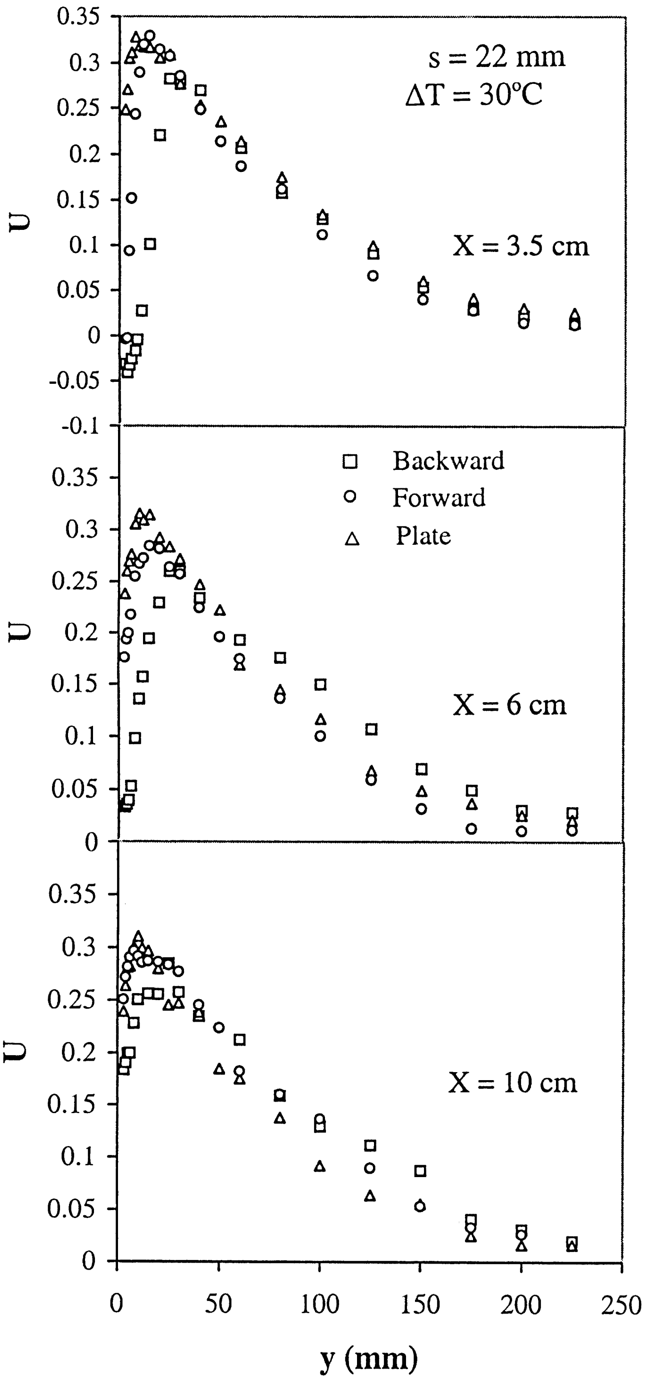

Fig. 4. Dimensionless mean streamwise velocity distributions.

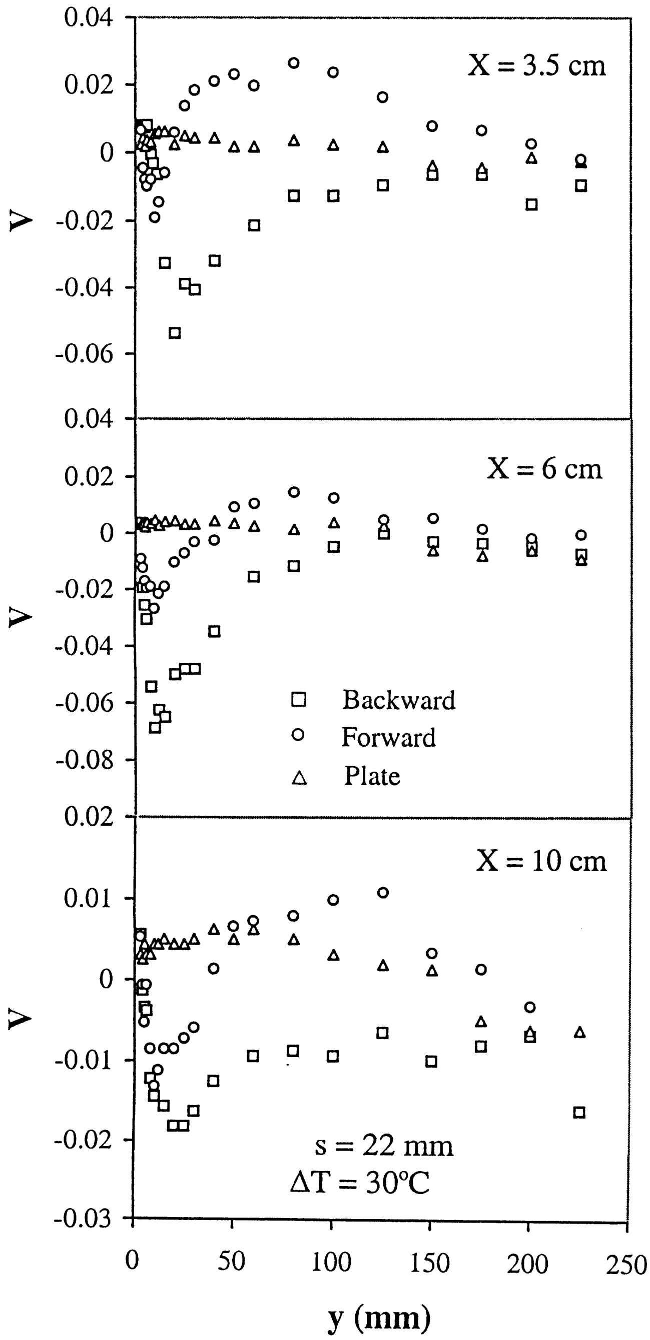

Fig. 5. Dimensionless mean transverse velocity distributions.

($x = 3.5, 6$, and 10 cm), for a backward-facing/forward-facing step height of 22 mm. It should be noted that $x = 0$ is the streamwise location at which the backward-facing and forward-facing steps are introduced into the flow geometry, as shown in Fig. 2. The resulting temperature difference between the heated surface and the free stream is 30 °C (corresponding to a local Grashof number $Gr_{xi} = 6.45 \times 10^{10}$).

The dimensionless time-mean streamwise and transverse velocity and temperature distributions at $x = 3.5, 6$, and 10 cm are presented in Figs. 4, 5, and 6, respectively. These figures illustrate the effect of backward-facing and forward-facing steps on the turbulent natural convection along a vertical heated flat plate. Fig. 4 shows that the time-mean streamwise velocity profiles at $x = 3.5$ exhibit negative mean streamwise velocity component for the cases of backward-facing and forward-facing steps. This is because the introduction of the backward-facing step or forward facing step in the flow geometry causes a separation at the upper sharp corner of the step creating a recirculating region downstream of the step. Also, as can be seen from the

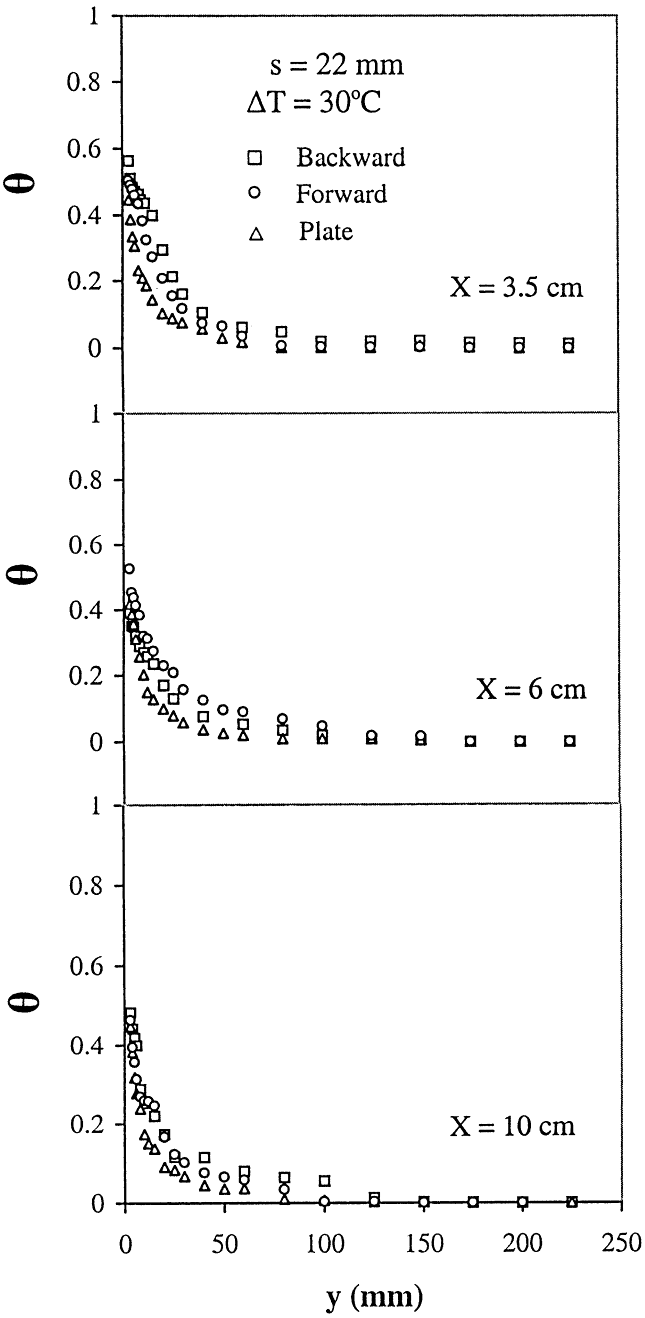

Fig. 6. Dimensionless mean temperature distributions.

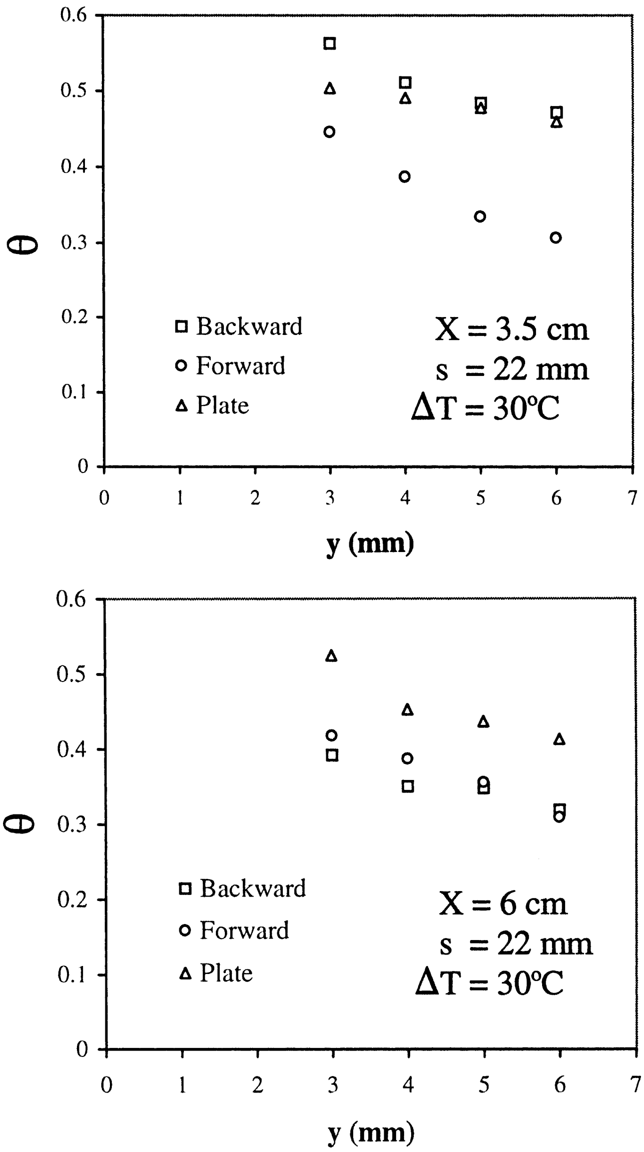

Fig. 6A.

figure, the magnitude of the negative streamwise velocity component is larger, in the negative sense, for the case of backward-facing step than that of the forward-facing step. For the case when a forward-facing step is introduced, it was observed that two recirculating regions develop, one in front or upstream and the other downstream from the step. It was determined that for the experimental conditions in this study, the length of the recirculating region that develops in front of the forward-facing step was very small with $x_s = 1.6$ cm or 0.727 of the step height, whereas the recirculating region that develops downstream of the step is very shallow with a reattachment length of $x_r = 4.2$ cm or 1.9 step heights. On the other hand, when a backward-facing step is introduced, it was observed that only one recirculating region develops and it is located behind the step. It was observed that the recirculating region associated with a backward-facing step is thicker and longer than the recirculating region associated with the forward-facing step. However, the reattachement length in the case of a backward-facing step is not steady, and it oscillates in the region between $5.5 < x_r < 6.5$ cm or 2.5–2.95 step heights. Measurements that are reported in this region represent the average value resulting from these oscillations. The figure also shows that the effect of both steps on the flow field continues to be present even at large distances downstream from the step ($x = 10$ cm). The uncertainty in the measurements are ±0.1 mm in y, ±1 mm in x, and ±3% in U.

Fig. 5 shows the effect of backward-facing and forward-facing steps on the dimensionless time-mean transverse velocity of turbulent natural convection along a vertical flat plate. It can be seen clearly from the figure that, for all cases (i.e., flat plate, backward-facing step, and forward-facing step), the time-mean transverse velocity in the flow region that is very close to the heated wall is always positive. In addition, all of the time-mean transverse velocity distributions exhibit negative velocity at the outer edge of the boundary-layer, indicating flow entrainment of the ambient air into the boundary-layer. Also, it can be seen from the figure that when a step is introduced in the flow geometry, the mean transverse velocity distributions exhibit negative velocity components in the region near the heated surface. The reason for the negative mean transverse velocity component, in the case of backward-facing step, is the sudden expansion in the flow geometry, which causes the streamlines to curve toward the downstream heated surface causing a recirculation region to develop behind the step. On the other hand, in the case of a forward-facing step, the reason for the negative time-mean transverse velocity in the region near the heated wall is the sudden compression in the flow geometry, which causes the streamlines to curve outward from the upstream heated wall near the step and then the streamlines are forced to curve towards the downstream heated wall causing a shallow recirculation region to develop downstream of the step. The figure clearly shows that the magnitude of the negative mean transverse velocity component in the region near the heated wall is larger, in the negative sense, for the case of backward-facing step than that of the forward-facing step. The figure also shows that the magnitude of the negative transverse velocity component in the flow region near the heated downstream wall decreases as the streamwise distance increases downstream from the step, indicating that the effect of the backward-facing/forward-facing step decreases as the streamwise distance increases downstream from the step. The uncertainty in the measured values for V is $\pm 3\%$.

The effect of the backward-facing and forward-facing steps on the non-dimensional local time-mean temperature distributions is shown in Fig. 6. The figure shows that the temperature distribution asymptotically approaches the mainstream temperature as the distance from the heated wall increases. It can be seen (from Fig. 6A) that at $x = 3.5$ cm, the magnitude of the temperature gradient normal to the heated surface is lower, for the case of the backward-facing step, and it is higher, for the case of forward-facing step, than that of the heated flat plate ($s = 0$). However, at $x = 6$ and 10 cm, the magnitude of the temperature gradient is higher (i.e., the heat transfer rate is higher) for both types of the step than that of the heated flat plate. The uncertainty in the measured values for θ is $\pm 2\%$.

Figs. 7, 8, and 9 illustrate the effects of forward-facing and backward-facing steps on the distributions of the turbulent intensities of streamwise and transverse velocity and temperature fluctuations. Figs. 7 and 8 show that the values of turbulent intensities of both streamwise and transverse velocity fluctuations at a streamwise location increase to a

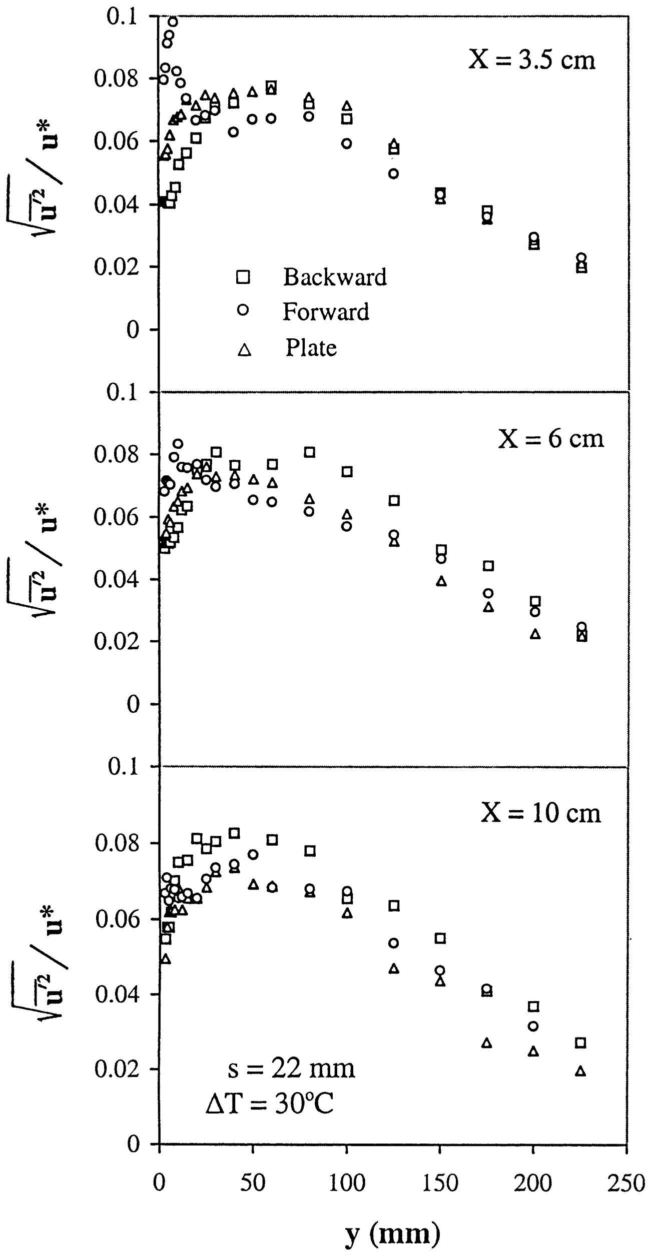

Fig. 7. Distributions of streamwise velocity fluctuation.

maximum as the distance from the heated wall increases, then start to decrease as the distance from the heated wall continues to increase, reaching a minimum value in the free stream. As can be seen from the figures, the maximum turbulent intensities of both the streamwise and the transverse velocity fluctuations at a streamwise location appear to occur at the same distance from the heated wall. The magnitudes of the turbulent intensities of streamwise velocity fluctuations are higher than those of the transverse velocity fluctuations. The figures clearly show that the intensities of turbulent fluctuations of both velocity component increase when a backward-facing or a forward-facing step is introduced to the flow geometry. This is because the step (backward-facing or forward-facing) acts as trigger, thus enhancing the

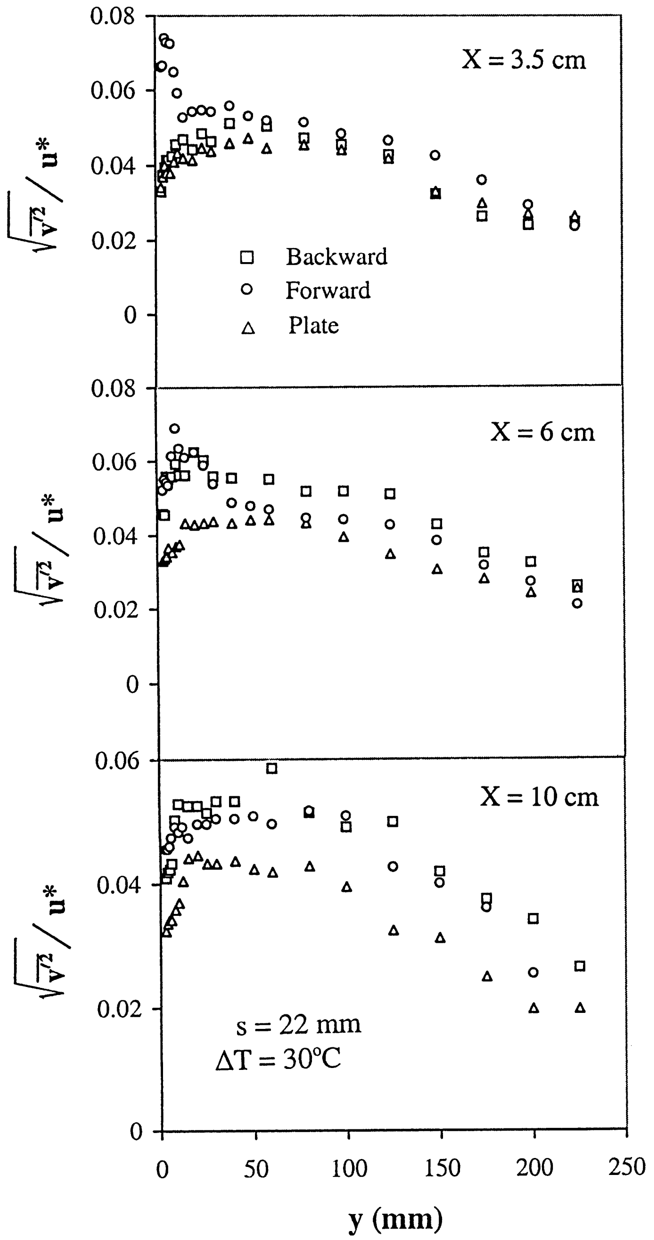

Fig. 8. Distributions of transverse velocity fluctuation.

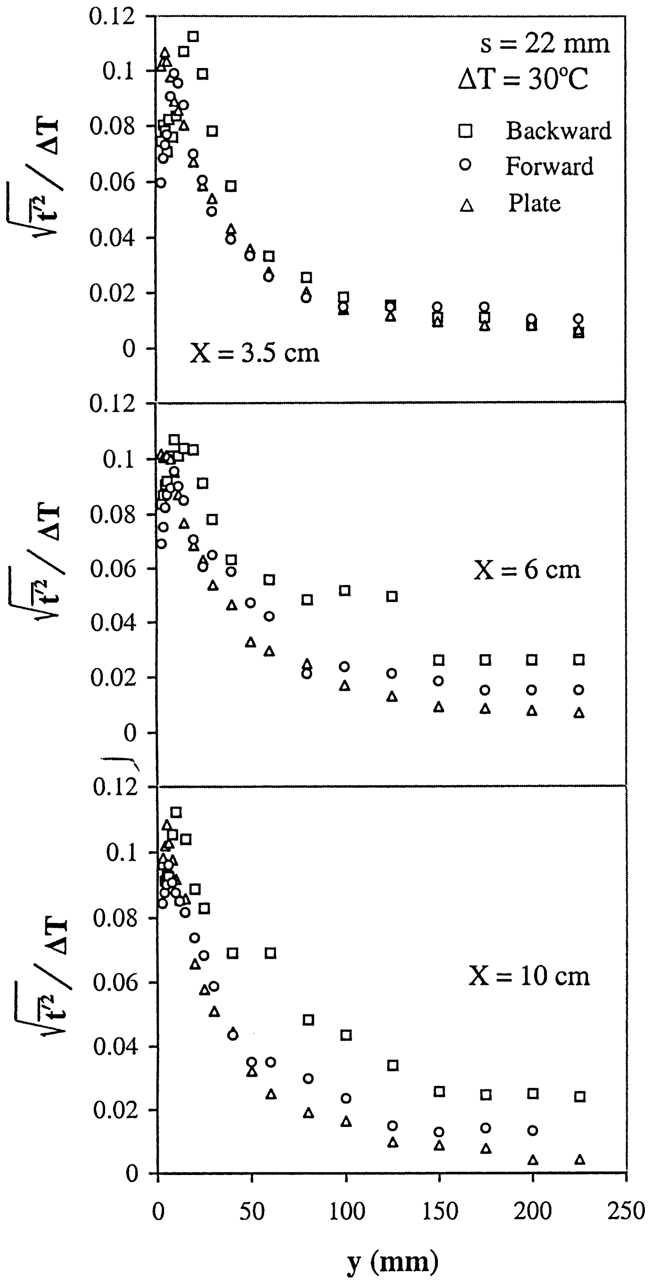

Fig. 9. Distributions of temperature fluctuation.

intensity of turbulent fluctuations. The distributions of the turbulent intensities of the temperature fluctuations (Fig. 9) exhibit a sharper peak than those of the turbulent intensities of velocity fluctuations. Also, the location of the maximum turbulent intensities of temperature fluctuations is closer to the heated wall than those of the turbulent intensities of velocity fluctuations. As can be seen from the figure, similar to the velocity fluctuations, the turbulent intensities of the temperature fluctuations increase when a step is introduced. The uncertainties in these measurements are ±6% in $\sqrt{\overline{u'^2}}/u^*$, and $\sqrt{\overline{v'^2}}/u^*$, and ±4% in $\sqrt{\overline{t'^2}}/\Delta T$.

The local convective heat transfer coefficients were determined from the measured temperature distributions (temperature gradients) in the laminar sublayer near the heated wall. For a given streamwise location, the air temperature was measured at four different locations within 0.5 mm from the heated wall in order to establish the temperature gradient at the heated wall. The temperature distributions in the laminar sublayer near the heated wall were linear and the temperature fluctuations in that region were near zero. The temperature measurements that were made in that region could not be included in Fig. 6 because of scale limitations. This technique for determining the surface temperature gradient and the local convective heat transfer coefficient in turbulent flow was utilized and validated by Tsuji and Nagano [14,15], Qiu et al. [16], and Abu-Mulaweh et al. [10]. The local Nusselt number was calculated from the measured temperature field

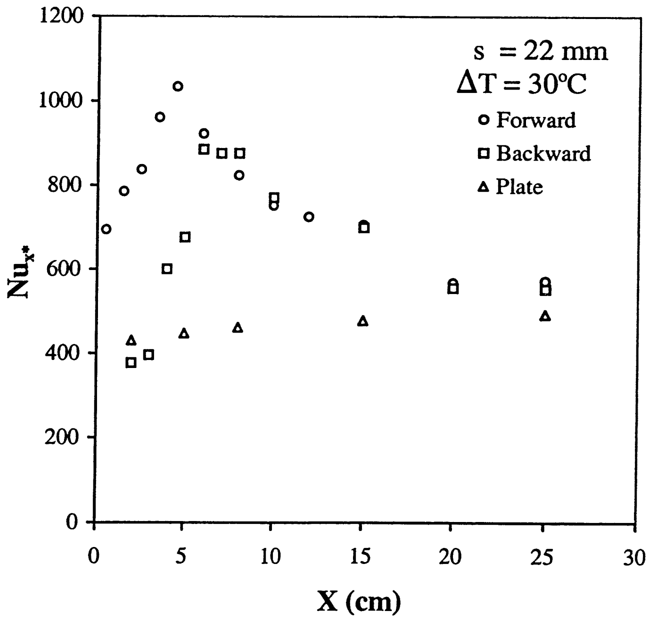

Fig. 10. Local Nusselt number variation.

using the following relation: $Nu_{x^*} = \frac{-\partial\theta}{\partial Y}|_{y=0}$. Fig. 10 illustrates the effect of backward-facing and forward facing steps on the local Nusselt number of turbulent natural convection along a vertical heated flat plate. The figure presents the variation of the experimentally deduced local Nusselt number downstream from the location of the step. When a step is introduced in the flow geometry, the Nusselt number increases with increasing distance from the step, reaching a maximum value in the vicinity of the reattachment region of almost twice that of the flat plate value for the case of backward-facing step and two and a half times that of the flat plate value for the case of forward-facing step. The impact of the relatively cooler fluid from the shear layer on the heated wall and the deflection of cooler fluid into the recirculating flow region downstream of the step cause this rapid increase in the Nusselt number. In the case of backward-facing step, the local Nusselt number value is smaller than that of the flat plate in the region close to the step. This agrees with the results shown in Fig. 6A, where at $x = 3.5$ cm, the magnitude of the temperature gradient normal to the heated surface for the case of backward-facing step is lower than that of the flat plate. On the other hand, the local Nusselt number for the case of forward-facing step in this region is much higher (i.e., the heat transfer is much higher) than both the flat plate and the backward-facing step. The magnitude of the local Nusselt number decreases as the distance continues to increase in the streamwise direction and asymptotically approaches the flat plate value. The measured local Nusselt number for the case of flat plate compares well (within 3%) with results predicted by empirical correlations reported in the literature, such as $Nu_{x^*} = 0.11\ Gr_{x^*}^{1/3}$ that was reported by Warner and Arpaci [17]. The uncertainty in x is ± 1 mm and in the measured Nusselt number is $\pm 6\%$.

4. Conclusions

(1) The introduction of a backward-facing step or forward facing step in the flow geometry on turbulent natural convection along a vertical heated flat plate affects the flow and thermal fields significantly.
(2) When a forward-facing step is introduced, the present results reveal that the maximum Nusselt number, which is approximately two and a half times that of a flat plate value at similar flow and thermal conditions, occurs in the vicinity of the reattachment region. For the experimental conditions, the size of the recirculating flow region upstream of the step is 1.6 cm or 0.767 of a step height and the reattachment length of the recirculation region downstream of the step is 4.2 cm or 1.9 of a step height.
(3) When a backward-facing step is introduced, the maximum Nusselt number, which is approximately twice that of a flat plate value at similar flow and thermal conditions, also occurs in the vicinity of the reattachment region. For the experimental conditions, the reattachment region is not steady and its center varies between 5.5 and 6.5 cm or 2.5–2.95 step heights.

References

[1] H.I. Abu-Mulaweh, B.F. Armaly, T.S. Chen, Measurements of laminar mixed convection flow over a horizontal forward-facing step, J. Thermophys. Heat Trans. 7 (1993) 569–573.
[2] J.T. Lin, B.F. Armaly, T.S. Chen, Mixed convection in buoyancy-assisting vertical backward-facing step flows, Internat. J. Heat Mass Trans. 33 (1990) 2121–2132.
[3] B. Hong, B.F. Armaly, T.S. Chen, Laminar mixed convection in a duct with a backward-facing step—the effects of inclination angle and prandtl number, Internat. J. Heat Mass Trans. 36 (1993) 3059–3067.
[4] H.I. Abu-Mulaweh, B.F. Armaly, T.S. Chen, Laminar natural convection flow over a vertical backward-facing step, J. Heat Trans. 117 (1995) 895–901.
[5] H.I. Abu-Mulaweh, B.F. Armaly, T.S. Chen, Laminar natural convection flow over a vertical forward-facing step, J. Thermophys. Heat Trans. 10 (1996) 517–523.
[6] J.C. Vogel, J.K. Eaton, Combined heat and fluid dynamics measurements downstream of a backward-facing step, J. Heat Trans. 107 (1985) 922–929.
[7] K. Abe, T. Kondoh, Y. Nagano, A new turbulence model for predicting fluid flow and heat transfer in separating and reattaching flows—II. Thermal field calculations, Internat. J. Heat Mass Trans. 38 (1994) 1467–1481.
[8] G.H. Rhee, H.J. Sung, A Nonlinear low-Reynolds-number k–ε model for turbulent separated and reattaching flows—II. Thermal field calculations, Internat. J. Heat Mass Trans. 39 (1996) 3465–3474.
[9] T. Inagaki, Heat transfer and fluid flow of turbulent natural convection along a vertical flat plate with a backward-facing step, Experimental Heat Trans. 7 (1994) 285–301.
[10] H.I. Abu-Mulaweh, T.S. Chen, B.F. Armaly, Turbulent natural convection flow over a vertical backward-facing step, Experimental Heat Trans. 12 (1999) 295–308.
[11] R.J. Moffat, Describing the uncertainties in experimental results, Experimental Thermal Fluids Sci. 1 (1988) 3–17.
[12] R. Cheesewright, M.H. Mirzai, The correlation of experimental velocity and temperature data for a turbulent natural convection

boundary layer, in: Proc. 2nd UK National Conference on Heat Transfer, 1988, pp. 79–89.

[13] O.A. Plumb, L.A. Kennedy, Application of a K–ε turbulence model to natural convection from a vertical isothermal surface, J. Heat Trans. 99 (1977) 79–85.

[14] T. Tsuji, Y. Nagano, Characteristics of a turbulent natural convection boundary layer along a vertical flat plate, Internat. J. Heat Mass Trans. 31 (1988) 723–1734.

[15] T. Tsuji, Y. Nagano, Turbulence measurements in natural convection boundary layer along a vertical flat plate, Internat. J. Heat Mass Trans. 31 (1988) 2101–2111.

[16] S. Qiu, T.W. Simon, R.J. Volino, Evaluation of local wall temperature, heat flux, and convective heat transfer coefficient from the near-wall temperature profile, in: ASME Proceedings of the 30th Natl. Heat Transfer Conf., HTD, Vol. 318, 1995, pp. 45–52.

[17] C.Y. Warner, V.S. Arpaci, An investigation of turbulent natural convection in air at low pressure along a vertical heated flat plate, Internat. J. Heat Mass Trans. 11 (1968) 397–406.